Mem. Amer. Math. Soc. **315**, 1985]. Approximately one-third of the entries are q-continued fractions. Several of these are related to Ramanujan's only published continued fraction, the famous Rogers–Ramanujan continued fraction. However, most of the entries are new. For each entry the authors give a proof and/or provide a reference to a proof. They also discuss connections with other entries and with the works of others.

This is a scholarly work that requires careful reading and checking in order to be fully appreciated. However, there is nothing too technical to prevent it from being read by someone with little or no knowledge of continued fractions. Both the expert and non-expert will profit from studying its contents and will be sure to become Ramanujan fans. It is recommended reading for anyone interested in special functions or approximations and expansions.

DAVID R. MASSON

Y. XU, *Common Zeros of Polynomials in Several Variables and Higher Dimensional Quadrature*, Pitman Research Notes in Mathematics Series **312**, Longman Scientific & Technical, Essex (U.K.), 1994.

This book collects some significant recent results in the theory of multidimensional quadrature formulas. Given a square positive functional $\mathscr{L}(f)$ on the space of the multivariate polynomials, a quadrature formula is an approximation of this functional of the form $\sum_{k=1}^{N} \lambda_k f(\mathbf{x}_k)$ that is exact for all polynomials f up to a certain degree. The nodes $\mathbf{x}_k$ are restricted to be real and the weights λ_k to be positive.

As in the one-dimensional case, the theory of quadrature formulas is based on the theory of orthogonal polynomials. The author uses a compact vector notation for the orthogonal polynomials and in a preliminary chapter he reviews their theory. The nodes of the quadrature formula are described as the common zeros of a set of quasi-orthogonal multivariate polynomials, which depend on matrices of parameters. Rather than relying on algebraic geometry for studying the common zeros of these polynomials, the author constructs properly tailored truncated block Jacobi matrices so that the common zeros of the polynomials are joint eigenvalues of these matrices. This leads to a characterization of the sets of quasi-orthogonal polynomials that generate interpolatory quadrature formulas with real nodes and positive weights. This characterization takes the form of nonlinear matrix equations in the parameters of these sets. Theorems 4.1.4 and 7.1.4 are, however, not properly formulated so that the reader may wrongly think that it is a characterization of the sets of polynomials whose common zeros are all real. The approach of the author is general; it deals with quadrature formulas of odd degree as well as of even degree. The relation with Möller's lower bound is explored in a separate chapter. Another chapter illustrates the theory with examples for which the nonlinear matrix equations can be solved.

The book is a research paper recommended to persons interested in the theoretical aspects of multidimensional quadrature and in multivariate orthogonal polynomials. It is self-contained and assumes no specific knowledge.

PIERRE VERLINDEN

K. KITAHARA, *Spaces of Approximating Functions with Haar-like Conditions*, Lecture Notes in Mathematics **1576**, Springer-Verlag, 1994, x + 110 pp.

The book contains five chapters and two appendices, with each chapter containing a problems section. A considerable portion of the book is based on the author's own work, some of which has not yet appeared elsewhere.

Chapter 1 introduces the Haar-like conditions mentioned in the title, and the definitions of $H_{\mathscr{F}}$, $T_{\mathscr{F}}$, and $WT_{\mathscr{F}}$-systems, where $\mathscr{F}$ is a set of n-tuples of linear functionals. Haar, Chebyshev, and weak Chebyshev systems are particular cases, obtained by choosing $\mathscr{F}$ to be an appropriate set of n-tuples of point evaluation functionals. These definitions are so general that, as the author points out, any system of linearly independent elements of a real vector space E is an $H_{\mathscr{F}}$-system for some $\mathscr{F}$. This chapter also gives examples of these systems and the spaces they generate, and discusses various relationships among them. The examples discussed include interpolating spaces, spaces of step-functions, and various classes of Chebyshev and weak Chebyshev systems. This chapter also introduces spaces that will be discussed in later chapters. For instance Stečkin's AC-spaces, i.e., the subspaces G of a normed space E, having the property that those elements of E that do not have a unique element of best approximation in G form a set of the first category in E, and $C_0(X)$, which is the set of all functions continuous in the Hausdorff space X, having the property that for each $\varepsilon > 0$, the set $\{x \in X : |f(x)| \geqslant \varepsilon\}$ is compact.

Chapter 2 discusses characterizations of various classes of AC, Chebyshev, and weak Chebyshev spaces defined in $C[a, b]$, $C(\mathbb{R})$ or $C_0(Q)$, where Q is a locally compact subset of $\mathbb{R}$. All these spaces are endowed with the norm of the supremum, and the characterizations that are discussed include properties of elements of best approximation, length of alternations, existence of interpolations, and decompositions of $\mathbb{R}^2$, among others. The material presented includes work of Deutsch *et al.*, Jones and Karlowitz, Feinerman and Newman, Blatter, Morris and Wulbert, Stockenberg, and the author.

Chapter 3 studies properties of Chebyshev and weak Chebyshev spaces defined on compact intervals. The topics discussed include properties of spaces of derivatives, relationships between AC spaces and oscillation spaces, integral representation of ECT and Markov systems (without proofs), existence of adjoined functions, best L^1 approximation, and density of infinite Markov systems. Unfortunately, the recent and important work of P. Borwein *et al.* (cf. *J. Approx. Theory* **63** (1990), 56–64) on the last-named topic, is not mentioned. The material presented in this chapter is based mostly on work of Cheney and Wulbert, Gierz and Shekhtman, Hobby and Rice, Karlin and Studden, Micchelli, Pinkus, Kroó, Schmidt and Sommer, Strauss, Zielke, Zwick, the author, and the reviewer. The proof of Theorem 3.3.5 can be shortened by using a result of the reviewer (*J. Approx. Theory* **22** (1978), 356–359).

Chapter 4 studies approximation by monotone increasing or convex functions defined on intervals, with values in an ordered Banach space. Topics discussed include properties of spaces of functions of bounded variation, and best approximation by monotone increasing or convex functions, including existence and uniqueness of best approximants, and conditions that guarantee the existence of continuous or differentiable best approximants. These results depend on the type of Banach space considered, which may be reflexive, uniformly convex, Hilbert, etc. This chapter includes work of Barbu and Precupanu, Darst and Huotari, Darst and Sahab, Kamthan and Gupta, and the author.

Chapter 5 deals with approximation by spaces of step functions. Jackson-type theorems are obtained. The material in this chapter is based on work of Feinerman, Feinerman and Newman, and the author. Appendix 1 (Dirichlet tilings) and Appendix 2 (minimum diameter property) are related to topics discussed in this chapter.

The author is to be commended for presenting in coherent fashion many results that had never appeared in book form, and for his effort to give credit to the many authors whose work he cited.

RICHARD A. ZALIK

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